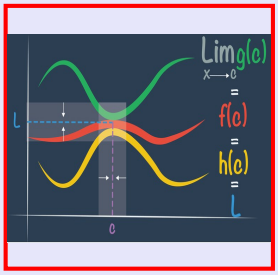
Math 261

Fall 2023

Lecture 20



Feb 19-8:47 AM

Evaluate $\lim_{x \to \pi/4} \dfrac{\tan x - 1}{x - \pi/4} = \dfrac{\overset{\to 1}{\tan \pi/4} - 1}{\pi/4 - \pi/4} = \dfrac{0}{0}$ I.F.

Hint: $f(x) = \tan x$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f\left(\frac{\pi}{4}\right) = \tan \pi/4 = 1$$

$$\lim_{x \to \pi/4} \frac{f(x) - 1}{x - \pi/4} = \lim_{x \to \pi/4} \frac{f(x) - f(\pi/4)}{x - \frac{\pi}{4}} = f'\left(\frac{\pi}{4}\right)$$

$$= \sec^2\left(\frac{\pi}{4}\right)$$

$$= \left(\frac{1}{\cos \pi/4}\right)^2$$

$$= \left(\frac{1}{\frac{\sqrt{2}}{2}}\right)^2$$

$$= \left(\frac{2}{\sqrt{2}}\right)^2$$

$$= \frac{4}{2} = \boxed{2}$$

Since $f(x) = \tan x$

$f'(x) = \sec^2 x$

Oct 2-11:28 AM

$f(x) = \cot x$, find $f'(x)$

$$\frac{d}{dx}[f(x)] = \frac{d}{dx}[\cot x] = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right]$$

$$= \frac{\frac{d}{dx}[\cos x]\cdot \sin x - \cos x \cdot \frac{d}{dx}[\sin x]}{[\sin x]^2} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-1\,[\underbrace{\sin^2 x + \cos^2 x}_{1}]}{\sin^2 x} = \frac{-1}{\sin^2 x} = \boxed{-\csc^2 x}$$

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\boxed{\begin{array}{l} \frac{d}{dx}[\sec x] \\ \frac{d}{dx}[\csc x] \end{array}}$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

Oct 3-10:31 AM

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

$$\frac{d}{dx}[f(x) \cdot g(x)] = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2}$$

Oct 3-10:37 AM

Power Rule:

$$\frac{d}{dx}\left[x^n\right] = n x^{n-1}$$ n is a positive integer

$$f(x) = x^n$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^n} + n x^{n-1} h + \frac{n(n-1)}{2!} x^{n-2} h^2 \cdots + h^n - \cancel{x^n}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}\left[n x^{n-1} + \frac{n(n-1)}{2!} x^{n-2} h + \cdots + h^{n-1}\right]}{\cancel{h}}$$

$$= \lim_{h\to 0} \left[n x^{n-1} + \frac{n(n-1)}{2!} x^{n-2} \cancel{h}^{\to 0} + \cdots + \cancel{h^{n-1}}^{\to 0}\right]$$

$$= \boxed{n x^{n-1}}$$

$$\boxed{\frac{d}{dx}\left[x^n\right] = n x^{n-1}}$$

$$\frac{d}{dx}\left[x^8\right] = 8x^{8-1} = \boxed{8x^7}$$

$$\frac{d}{dx}\left[x^1\right] = 1 \cdot x^{1-1}$$
$$= 1 \cdot x^0 = 1 \cdot 1$$
$$= \boxed{1}$$

$$\frac{dx}{dx} = 1$$

Oct 3-10:40 AM

Find $\frac{d}{dx}\left[x^2 + \sin x\right]$

$$= \frac{d}{dx}\left[x^2\right] + \frac{d}{dx}\left[\sin x\right]$$

$$= 2x^{2-1} + \cos x = \boxed{2x + \cos x}$$

Find $\frac{d}{dx}\left[x^3 \cos x\right]$

$$= \frac{d}{dx}\left[x^3\right] \cdot \cos x + x^3 \cdot \frac{d}{dx}\left[\cos x\right]$$

$$= 3x^2 \cos x + x^3 \cdot -\sin x$$

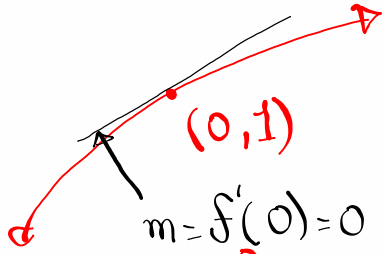
$$= 3x^2 \cos x - x^3 \sin x$$

Oct 3-10:49 AM

Find eqn of the tan. line to the graph of

$f(x) = \dfrac{2}{1+\cos x}$ at $x=0$.

$f(0) = \dfrac{2}{1+\cos 0} = \dfrac{2}{1+1} = \dfrac{2}{2} = \boxed{1}$



$y-1 = 0(x-0)$

$\boxed{y=1}$

$m = f'(0) = 0$

$$f'(x) = \frac{\overset{0}{\cancel{\frac{d}{dx}[2]}}\cdot(1+\cos x) - 2\cdot\frac{d}{dx}[\overset{0}{\cancel{1}}+\overset{-\sin x}{\cancel{\cos x}}]}{(1+\cos x)^2}$$

$f'(x) = \dfrac{2\sin x}{(1+\cos x)^2}$

$f'(0) = \dfrac{2\sin 0}{(1+\cos 0)^2} = \dfrac{0}{4} = \boxed{0}$

Oct 3-10:53 AM

Prove $\dfrac{d}{dx}[c] = 0$

Let $f(x) = C$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{C-C}{h} = \lim_{h\to 0}\frac{0}{h} = \lim_{h\to 0} 0 = 0$$

$f(x) = C$ $\qquad$ $f'(x) = 0$

Prove $\dfrac{d}{dx}[c f(x)] = c f'(x)$

$$\frac{d}{dx}[c f(x)] = \lim_{h\to 0}\frac{c f(x+h) - c f(x)}{h}$$

$$= \lim_{h\to 0}\frac{c[f(x+h)-f(x)]}{h}$$

$$= c\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= c f'(x)$$

Oct 3-11:01 AM

Find $\frac{d}{dx}\left[\frac{1}{2}x^4 - \frac{1}{3}x^3 + 10\right]$

$$= \frac{d}{dx}\left[\frac{1}{2}x^4\right] - \frac{d}{dx}\left[\frac{1}{3}x^3\right] + \frac{d}{dx}[10]$$

$$= \frac{1}{2}\frac{d}{dx}[x^4] - \frac{1}{3}\frac{d}{dx}[x^3] + \frac{d}{dx}[10]$$

$$= \frac{1}{2}\cdot 4x^3 - \frac{1}{3}\cdot 3x^2 + 0$$

$$= \boxed{2x^3 - x^2}$$

Oct 3-11:06 AM

Given $f(x) = 4x^7 - 5x^3 + 2x$

1) find $f(0) = 4(0)^7 - 5(0)^3 + 2(0) = \boxed{0}$

2) find $f'(x) = 4\cdot 7x^6 - 5\cdot 3x^2 + 2\cdot 1$

$$= \boxed{28x^6 - 15x^2 + 2}$$

3) find $f'(0) = 28(0)^6 - 15(0)^2 + 2 = \boxed{2}$

4) Eqn of tan. line at $x=0$.

$$y - 0 = 2(x-0) \rightarrow y = 2x$$

5) Eqn of normal line at $x=0$.

$$y - 0 = \frac{-1}{2}(x-0) \rightarrow y = \frac{-1}{2}x$$

Oct 3-11:12 AM

At which points does the graph of $f(x)=\frac{x}{x^2+9}$ have horizontal tangent line?

$(3, f(3))$
$(-3, f(-3))$

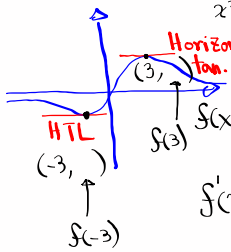
Domain $\to (-\infty, \infty)$ NO V.A.

$f(-x)=\frac{-x}{(-x)^2+9}=\frac{-x}{x^2+9}=-\frac{x}{x^2+9}=-f(x)$

odd function
symmetric with respect to the origin

$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{x}{x^2+9}=\frac{\infty}{\infty}$ I.F.

$=\lim_{x\to\infty}\frac{\frac{x}{x^2}}{\frac{x^2}{x^2}+\frac{9}{x^2}}=\lim_{x\to\infty}\frac{\cancel{\frac{1}{x}}\to 0}{1+\cancel{\frac{9}{x}}\to 0}=\frac{0}{1}=\boxed{0}$ H.A.



$\to m=0 \to f'(x)=0$

$f(x)=\frac{x}{x^2+9}$

$f'(x)=\frac{1(x^2+9)-x\cdot 2x}{(x^2+9)^2}=\frac{9-x^2}{(x^2+9)^2}$

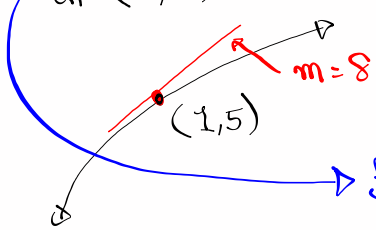
$f'(x)=0 \to 9-x^2=0 \to \boxed{x=\pm 3}$

Oct 3-11:18 AM

Find values for a and b if tan. line to $y=ax^2+bx$ at $(1,5)$ has slope of 8.

$f(x)=ax^2+bx$ $\qquad f'(1)=8$

at $(1,5) \to f(1)=5$



$f(1)=a(1)^2+b(1)$

$=\boxed{a+b=5}$

$\to f'(x)=2ax+b$

$f'(1)=2a(1)+b$

$=\boxed{2a+b=8}$

Solve

$-1\begin{cases} a+b=5 \\ 2a+b=8 \end{cases} \Rightarrow \begin{cases} -a-b=-5 \\ 2a+b=8 \end{cases}$

$\boxed{a=3}$

$\to 3+b=5$

$\boxed{b=2}$

Oct 3-11:28 AM